INTRODUCTION TO ROCKET PROPULSION

L1: Working Principles of Rocket Propulsion

How do rockets actually work?
In this lecture, we will define *rocket propulsion* and distinguish it from *jet propulsion*. We will also define *outer space* and investigate why rockets are needed to get there. We will review Newton's Laws of Motion and the concept of *momentum exchange*, which is the main working principle of rocket propulsion. Finally, we will estimate the thrust of a rocket that expels discrete masses from a space vehicle.

LEARNING GOALS:

1. Define rocket propulsion and explain the difference between rocket propulsion and jet propulsion.
2. Describe why rocket propulsion is needed to get to and travel around outer space.
3. Use the concept of momentum exchange to explain how rockets produce thrust force.
4. Calculate the final velocity of a rocket after a discrete piece of matter is expelled from the vehicle.

Figure 1: A turbofan jet engine. Check out this **YouTube video** to learn more about how jet engines work!

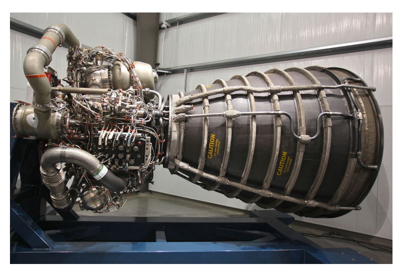

Figure 2: A chemical rocket engine. This engine is one of three that were used on the Space Shuttle.

WHY DO WE NEED ROCKETS?

To address this question, we'll need some definitions:

DEFINITION 1.1 The act of changing the motion of a body is called **propulsion**.

DEFINITION 1.2 **Jet propulsion** is a means of locomotion whereby a reaction force is imparted to a vehicle by the momentum of ejected matter.

DEFINITION 1.3 **Rocket propulsion** is a class of jet propulsion that provides thrust by ejecting matter, called the **propellant**, which is stored in the flying vehicle.

We need rockets because they allow us to go to and travel around outer space. Where exactly is *outer space*?

DEFINITION 1.4 **Outer space** is the region beyond Earth's atmosphere. The line between Earth's atmosphere and *outer space* is called the **Káráman line**, which is 100 kilometers (62 miles) above sea level.

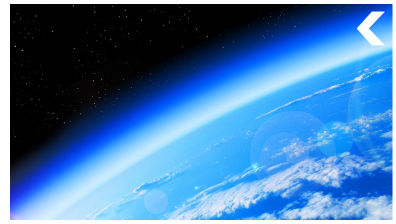

Figure 3: Earth's atmosphere viewed from space. Check out **this video** to learn more about the layers of Earth's atmosphere and where "outer space" begins.

At the altitude of the Káráman line, the atmosphere is so diffuse that *the air pressure is 3 million times less than the pressure at Earth's surface*. Humans definitely cannot breath in outer space! You might be surprised to learn that aircraft can't "breathe" in space either. Jet engines rely on the rich oxygen content of Earth's lower atmosphere to operate. Diatomic oxygen, also known as O_2, is an essential ingredient for the *chemical combustion reactions* that enable jet engines to work.

DEFINITION 1.5 A reaction in which fuel and oxidizer propellants are mixed together, resulting in an exothermic, or heat-releasing, reaction is called a **combustion reaction**.

Jet aircraft carry fuel, typically kerosene, which is injected into the combustion region of the engine. The vaporized fuel is then mixed with pressurized air from the atmosphere, which is rich in oxygen. The mixture is ignited so that the fuel and oxygen combust and burn. The combustion of fuel and oxygen creates hot gases that are expelled from the jet engine at high velocities. Without sufficient atmospheric oxygen, jet engines cannot function. Therefore, to get to outer space, we need a different approach!

Chemical rocket engines operate in a similar way to jet engines in that fuel and oxidizer are mixed to create hot gas that is expelled from the vehicle. However, rockets are different because they carry *both* the fuel and oxidizer on board. Rockets don't get their oxygen from the atmosphere like jet engines do. Therefore the operation of rocket engines is *independent of the atmospheric pressure*, although the performance does vary slightly, as we will see later on in the course.

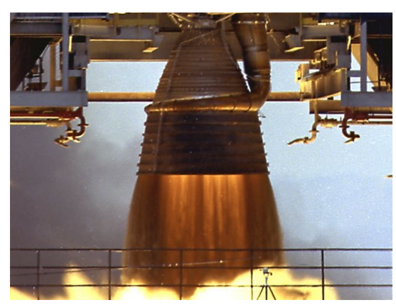

Figure 4: A chemical rocket engine uses fuel and oxidizer as propellant, which combust when mixed together. The reaction products are hot gases that are expelled from the nozzle at supersonic speeds.

Another really cool thing about rockets is that the flight speed of a rocket, meaning how fast the vehicle travels through space, is *independent of the velocity of the exhaust gases*. This is not the case for jet engines, for which the maximum vehicle speed is limited to velocity of the exhaust. Even if jets could operate at low atmospheric pressures, they still would not be able to reach high enough velocities to escape Earth's gravity. This is because the maximum exhaust velocity for a typical chemical combustion jet engine is much less than the velocity required to escape the Earth. Another drawback of jet engines is that the thrust decreases with flight speed, which makes it more challenging to go fast.

In summary, *rockets can get us to space* because they can operate in the vacuum of space and their performance is independent of the vehicle speed. With rockets, we can go fast! Really fast! We can escape Earth's gravity and even the Sun's gravity. We just need enough propellant to get us to where we want to go, which we will soon learn is not a trivial problem. But, before we dive into that, let's review the fundamental working principles of rocket propulsion.

WORKING PRINCIPLES OF ROCKETS

Rocket propulsion, like all forms of jet propulsion, relies on *momentum exchange*.

> DEFINITION 1.6 **Momentum exchange** is a transfer of momentum that occurs when two bodies collide or separate. The transfer results from the conservation of energy and the conservation of momentum.

In the case of a rocket, two bodies are separating from each other: the exhaust and the vehicle. The rocket engine exerts a force on the exhaust, which accelerates it away from the vehicle. The exhaust has momentum, so for the momentum of the vehicle-exhaust system to be conserved, the momentum of the vehicle must change. As a result, the vehicle is accelerated in the opposite direction of the exhaust.

Later in this section, we will consider an example that demonstrates the principle of momentum exchange. We'll calculate the change in velocity of a vehicle that expels a discrete piece of matter. We'll also calculate the thrust force that arises from expelling a discrete piece of matter. Before we jump into examples, let's review *Newton's Laws of Motion*, since they are essential for analyzing rockets.

Newton's Laws of Motion

1. A body in motion stays in motion. A body at rest stays at rest, unless acted upon by an external force.
2. The sum of the external forces acting on a body is equal to the time rate of change of momentum of the body. This law can be mathematically represented as:

$$\sum \vec{F} = \frac{d\vec{p}}{dt} \tag{1}$$

where $\vec{F}$ is an external force and $\vec{p}$ is the momentum of the body. We use $\vec{F} = m\vec{a}$ when the mass of the body is constant.
3. Every action has an equal and opposite reaction.

The Second and Third Laws of Motion are most relevant to rocket propulsion. When a rocket expels exhaust, it exerts a force on the exhaust. By the Third Law of Motion, the exhaust exerts an equal and opposite force on the rocket vehicle. This force is often called *thrust*. The thrust force can be calculated using the Second Law of Motion to determine the change in momentum of the rocket vehicle.

Before we consider a rocket that steadily discharges exhaust at a given rate, let's investigate what happens when a single chunk of matter is ejected from the vehicle. In this scenario, we can clearly define two states:

1. Initial state: *Before* the matter is expelled from the rocket.
2. Final state: *After* the matter is expelled from the rocket.

We can compare the energy and momentum of the system for these two states to determine the effect of momentum exchange. Let's do so in the following example!

Example 1:

Imagine a spaceship in outer space with zero velocity. The ship is far from any star or planet, so there are no gravitational forces. The astronauts on the ship are pretty bored, so they dig out their tennis rackets and start playing inside the ship. One of the astronauts hits the ball a little too hard and sends it flying out the back of the ship at a velocity of 235 kilometers per hour. If the mass of the ship is 10^6 kg and the mass of the ball is 57 g, what is final speed of the ship?

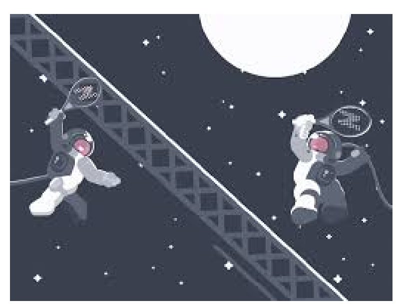

Figure 5: Tennis in space?! Yes, it has happened - check out **this video**!

Initial State

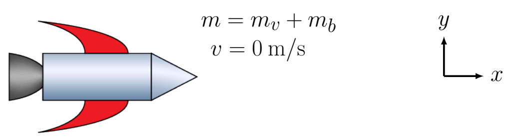

Figure 6: Initial state of the system.

Final State

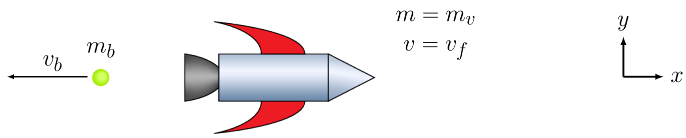

Figure 7: Final state of the system.

Let's think of the spaceship and the tennis ball as a system. Were any *external forces*, such as gravity, applied to the system? No. There were *internal forces*, such as the force the astronaut exerted on the tennis ball by hitting it with a racket and the equal and opposite force the tennis ball exerted on the racket. If there were no external forces applied to the system, then by Newton's Second Law we can conclude that the change in momentum of the system must be zero:

$$\sum \vec{F} = \frac{d\vec{p}}{dt} = 0 \tag{1.1}$$

This means that the momentum of the system *before* the tennis ball was hit is equal to the momentum of the system *after* serving the tennis ball. We can write this as:

$$\vec{p}_i = \vec{p}_f \tag{1.2}$$

where $\vec{p}_i$ is the initial momentum of the system and $\vec{p}_f$ is the final momentum of the system.

Figure 6 shows the initial state of the system. What is the initial momentum of the system? It's the initial momentum of the ship plus the initial momentum of the ball. Linear momentum is the product of the mass, m, of a body by the velocity, $\vec{v}$, of the body. Therefore, we can write the initial momentum of the system as:

$$\begin{aligned} \vec{p}_i &= \vec{p}_{s,i} + \vec{p}_{b,i} \\ &= m_s\,\vec{v}_{s,i} + m_b\,\vec{v}_{b,i} \end{aligned} \tag{1.3}$$

where s denotes quantities for the ship and b denotes quantities for the ball. The initial velocity of the ship and ball are the same because the two bodies are grouped together in the initial state. We can write this as:

$$\vec{p}_i = (m_s + m_b)\,\vec{v}_i \tag{1.4}$$

The initial velocity of the ship and ball is zero, so $\vec{p}_i = 0$.

Figure 7 shows the final state of the system. What is the final momentum of the system? We can calculate this in the same way we did in equation 1.3. The momentum of the system in the final state, after the ball has been ejected from the ship, is:

$$\begin{aligned} \vec{p}_f &= \vec{p}_{s,f} + \vec{p}_{b,f} \\ &= m_s\,\vec{v}_{s,f} + m_b\,\vec{v}_{b,f} \end{aligned} \tag{1.5}$$

Now let's apply equation 1.2, which is the conservation of momentum. We can set equation 1.4 equal to equation 1.5:

$$(m_s + m_b)\,\vec{v}_i = m_s\,\vec{v}_{s,f} + m_b\,\vec{v}_{b,f} \tag{1.6}$$

Note that we didn't plug in $\vec{v}_i = 0$ m/s just yet. We will solve for the final velocity of the ship in terms of the initial velocity, just in case we need to solve another problem where the initial velocity is non-zero.

Now we can solve for the final velocity of the ship:

$$\vec{v}_{s,f} = \vec{v}_i - \frac{m_b}{m_s}\,\vec{v}_{b,f} \tag{1.7}$$

Let's plug in some numbers! We know that $\vec{v}_i = 0\,\text{km/hr}\,\hat{x}$, $m_b = 0.059$ kg, $m_s = 10^6$ kg, and $\vec{v}_{b,f} = -235\,\text{km/hr}\,\hat{x}$.

Note that the $\hat{x}$ denotes the direction of the positive x-axis. When we see a $-\hat{x}$, that means the vector points in the negative x-direction.

$$\begin{aligned} \vec{v}_{s,f} &= 0\frac{\text{km}}{\text{hr}}\,\hat{x} - \frac{0.059\,\text{kg}}{10^6\,\text{kg}}\left(-235\,\frac{\text{km}}{\text{hr}}\,\hat{x}\right) \\ &= 1.39 \times 10^{-5}\,\frac{\text{km}}{\text{hr}}\,\hat{x} \\ &= 1.39\,\frac{\text{cm}}{\text{hr}}\,\hat{x} \end{aligned} \tag{1.8}$$

The final velocity of the ship is 1.39 cm/hr in the positive x-direction, which is not very fast at all. However, the action of serving a tennis ball out the back of a spaceship *did* cause the ship to move, which is pretty cool! We have now demonstrated the principle of *momentum exchange*.

As we saw in example 1, serving a tennis ball out of a spaceship isn't a particularly effective form of rocket propulsion. In the next example, let's estimate how many tennis balls we would need to accelerate the spaceship to a speed of 100 km/hr.

Example 2:

We know from example 1 that serving a single tennis ball from a 1 million kg spaceship results in a velocity increase of 1.32 cm/hr. Let's assume that the mass of the spaceship doesn't change very much after ejecting the tennis balls. This allows us to assume that the change in velocity induced by serving one tennis ball will be the same for any tennis ball. To obtain a rough estimate of the number of tennis balls needed to increase the speed to 100 km/hr, we can divide the desired change in velocity of the spaceship, Δv_s, by the change in velocity that results from serving a single tennis ball, Δv_b.

$$\begin{aligned} n &= \frac{\Delta v_s}{\Delta v_b} \\ &= \frac{100\,\text{km/hr}}{1.32 \times 10^{-5}\,\text{km/hr}} \\ &= 7.6 \times 10^6 \\ &= 7.6\,\text{million tennis balls!} \end{aligned} \tag{2.1}$$

Now wait a second, 7.6 million is a lot of tennis balls! That many tennis balls has a total mass of 433,000 kg, which is 43% of the mass of the spaceship. Remember we assumed that the mass of the spaceship stays approximately constant, but now we can see that this was not a good assumption!

It looks like we will need to take a different approach with this problem to account for how the mass of the spaceship changes. Let's use equation 1.7 to create an expression for the change in velocity of the spaceship that results from serving a single tennis ball:

$$\Delta v_s = -\frac{m_b}{m_s}\,\vec{v}_{b,f} \tag{2.1}$$

From our previous calculations, the Δv_s is 1.32 cm/hr for the first tennis ball. After the first tennis ball is expelled, we subtract 59 g from the mass of the spaceship and recalculate Δv_s. If you do the calculation, you'll find that Δv_s is still pretty much 1.32 cm/hr. That's because the spaceship mass only decreased by a tiny fraction. But what if we repeated this calculation again and again for millions of tennis balls? We certainly wouldn't want to do this by hand, but we can write a computer program do the hard work for us!

Using a computer program, we will find that 5.9 million tennis balls are needed to accelerate the spaceship to a velocity of 100 km/hr. Interestingly, this is less than our original estimate. Why? As the spaceship gets lighter, the change in velocity induced by serving a tennis ball increases. Take another look at equation 2.1 to convince yourself why this is true. The last tennis ball of the 5.9 million increases the velocity of the spaceship by 2.1 cm/hr.

ROCKET THRUST CALCULATION

In the previous section, we learn about the principle of momentum exchange. This helped us calculate the final speed of a rocket after a discrete piece of matter was expelled from the vehicle. In this section, we will calculate the *thrust force* that arises from the exchange of momentum by applying Newton's Second Law.

DEFINITION 1.7 The force that arises due to the momentum exchange of exhaust expelled from a rocket is called the **thrust force**. This is the force that accelerates the rocket-propelled vehicle in the opposite direction of the exhaust.

Example 3:

Continuing from previous examples, let's calculate the force exerted on the spaceship as a result of serving a tennis ball. If we knew how much force the ball was hit with, then we would know the force on the spaceship. However, all we know are the masses of the spaceship and tennis ball, the initial velocity of the system, and the final velocity of the tennis ball. We will also assume that the impact time between the racket and ball is 10 milliseconds.

For this problem, we will use Newton's Second Law, which states that the net force on a body is equal to the time rate of change of the momentum of the body. We'll apply Newton's Second Law to the tennis ball. Let's start by calculating the change in momentum of the tennis ball:

$$\begin{aligned}\Delta\vec{p}_b &= \vec{p}_{b,f} - \vec{p}_{b,i} \\ &= m_b\,\vec{v}_{b,f} - m_b\,\vec{v}_{b,i} \\ &= m_b\,\Delta\vec{v}_b\end{aligned} \tag{3.1}$$

where $\Delta\vec{v}_b = \vec{v}_{b,f} - \vec{v}_{b,i}$, which is the change in velocity of the ball. The initial velocity of the ball is zero, so the change in velocity of the ball is $\Delta\vec{v}_b = \vec{v}_{b,f}$. Therefore, the change in momentum of the tennis ball is $\Delta\vec{p}_b = m_b\vec{v}_{b,f}$.

Using Newton's Second Law, we can compute the force, $\vec{F}_b$, acting on the tennis ball:

$$\vec{F}_b = \frac{\Delta\vec{p}_b}{\Delta t} = \frac{m_b\,\vec{v}_{b,f}}{\Delta t} \tag{3.2}$$

To find the thrust force acting on the spaceship, $\vec{F}_s$, we can apply Newton's Third Law, which states that for every action there is an equal and opposite reaction:

$$\begin{aligned}\vec{F}_s &= -\vec{F}_s \\ &= -\frac{m_b\,\vec{v}_{b,f}}{\Delta t}\end{aligned} \tag{3.3}$$

Now let's plug in some numbers:

$$\begin{aligned}\vec{F}_s &= -\frac{m_b\,\vec{v}_{b,f}}{\Delta t} \\ &= -\frac{0.059\,\text{kg}(-235\,\text{km/hr}\,\hat{x})}{0.001\,\text{s}} \\ &= 339\,\text{N}\;\hat{x}\end{aligned} \tag{3.4}$$

The thrust force that arises from serving a tennis ball out the back of a spaceship is 339 Newtons in the positive x-direction. This force propels the ship in the positive x-direction.