INTRODUCTION TO ROCKET PROPULSION

L10: Electricity, Magnetism, and Plasmas

What are plasmas and how do we characterize them?
In this lecture, we will learn about plasmas! We'll start by reviewing concepts and equations fundamental to electricity and magnetism. We'll also learn about the motion of charged particles in electric and magnetic fields. Then we'll describe what plasmas are, how they're made, and which gases make the best plasmas. Finally, we'll learn about two parameters that are used to characterize plasmas: the plasma frequency and the debye length.

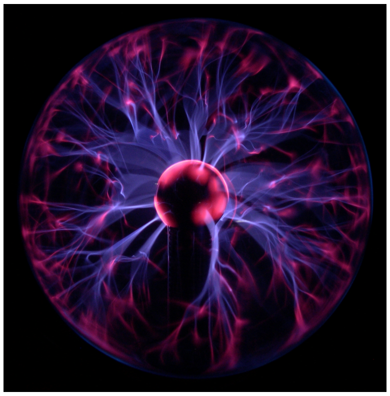

Figure 1: A plasma lamp.

LEARNING GOALS:

1. Explain the difference between forces that arise due to electric fields and those that arise due to magnetic fields.
2. Write the equations of motion for a charged particle in the presence of electric and magnetic fields.
3. Describe what a plasma is and explain what properties are desired for plasma thruster propellants.
4. Contrast the behavior of plasmas with that of neutral gases.
5. Compute the plasma frequency and debye length for a plasma.

INTRODUCTION TO ELECTRICITY AND MAGNETISM

Electrostatics

Let's consider two charged particles, shown in Figure 2. The electrostatic force that arises between these charged particles is given by Coulomb's law:

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1\, q_2}{{r_{12}}^2}\, \hat{\mathbf{r}}_{\mathbf{12}} \qquad (1)$$

where $\vec{F}_{12}$ is the force on the second charge due to the presence of the first charge, ϵ_0 is the permittivity of free space, q_1 is the charge of the first particle, q_2 is the charge of the second particle, and r_{12} is the magnitude of the vector $\vec{r}_{12}$, which is equal to the distance between the two charges. From Coulomb's law, we can observe that like charges repel and opposite charges attract.

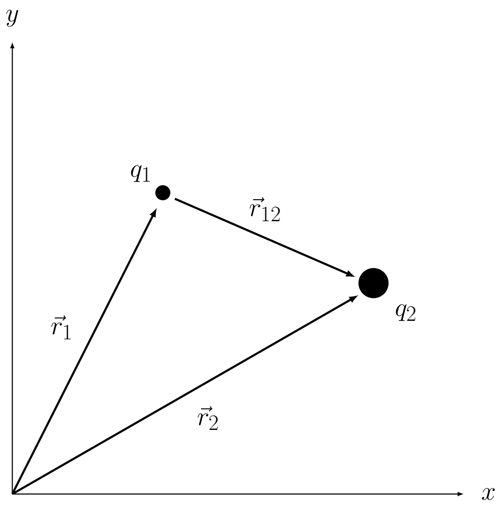

Figure 2: Locations of two charges in space.

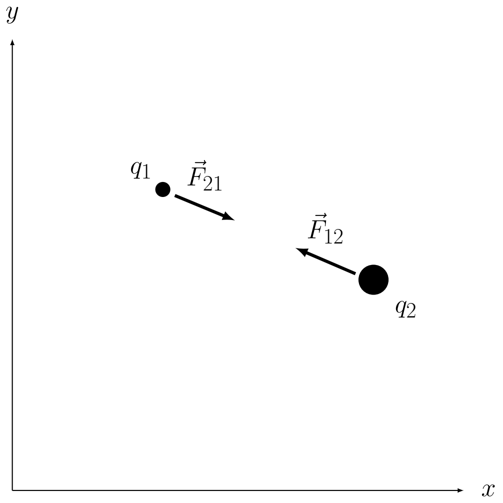

Figure 3: Attractive forces due to the electrostatic interaction between two charges in space. In this example we assume that q_1 and q_2 are oppositely charged.

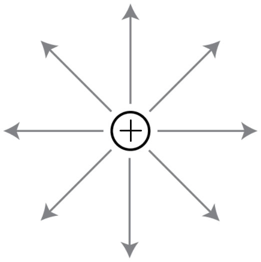

Figure 4: Electric field lines of a positive charge.

Charged particles create electric fields. The electric field created by a charged particle can be described mathematically by the following equation:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\,\frac{q}{r^2}\,\hat{\boldsymbol{r}} \tag{2}$$

where q is the charged of the particle, r is the distance from the particle, and $\hat{r}$ is the unit vector that points radially outward from the particle. Positive charges generate electric fields in which the field lines point radially outward, as shown in Figure 4. Negative charges generate electric fields in which the field lines point radially inward, as shown in Figure 5.

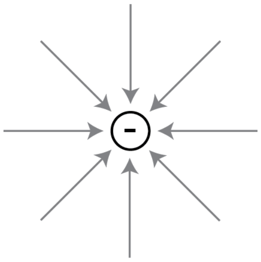

Figure 5: Electric field lines of a negative charge.

A charged particle will experience a force when it is in the presence of an electric field:

$$\vec{F} = q\vec{E} \tag{3}$$

where F is the force on the charged particle, q is the charge of the particle, and E is the electric field. Note: The units of electric field strength are Newtons/Coulomb, which is equivalent to volts/meter.

Electrostatic forces are conservative, which means that we can define a potential function, ϕ, such that:

$$\vec{E} = - \bigtriangledown \phi \tag{4}$$

where ϕ is the electric potential, which is measured in volts. Don't worry if you haven't seen the **gradient** symbol, $\bigtriangledown$, before. It's like a derivative and you'll see how it's used in the labs, if needed. If we consider an electric field in one dimension, then we can write the previous equation as:

$$E_x = -\frac{d\phi}{dx} \tag{5}$$

The electric potential generated by a charged particle is:

$$\phi = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \tag{6}$$

We can also define the electric potential energy, U_e, of a charged particle in the presence of an electric potential as:

$$U_e = q\phi \tag{7}$$

Magnetism

Magnetic fields are the counterpart of electric fields. Magnetic fields have two polarities: north and south. Unlike electric fields, magnetic fields always appear as *dipoles*. This means that there is no such thing as a single source of a north magnetic field, for example, which is called a magnetic monopole. Instead, a north field is always balanced by a south field, like in the bar magnet shown in Figure 6.

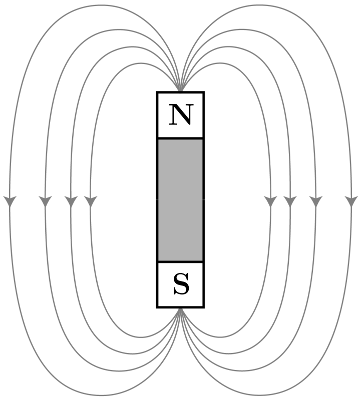

Figure 6: Magnetic field lines of a bar magnet.

Permanent magnets produce magnetic fields because of the spin alignment of the electrons in the atoms of the material. Magnetic fields can also be generated by moving charges, such as a wire that carries electrical current. We won't go into detail with this aspect of magnetism. The most important thing to know is that the magnetic force on a charged particle is given by the following equation:

$$\vec{F} = q\vec{v} \times \vec{B} \tag{8}$$

where $\vec{v}$ is the velocity of the charged particle and $\vec{B}$ is the magnetic field. Note that the × symbol indicates that this is a **cross-product** operation. Based on equation 8, we can see that stationary particles don't experience a force when in the presence of a magnetic field. Instead, only moving particles feel a force, and that force is proportional to their speed.

Fascinatingly, magnetic fields only accelerate particles through changing the direction of the particle's motion. The magnetic force is always perpendicular to the particle's velocity, which means that no work is done on the particle. Therefore, the speed of the particle remains constant and only the direction of the velocity changes.

Motion of Charged Particles in Electromagnetic Fields

Let's consider the trajectory of a charged particle in the presence of electric and magnetic fields. The net force the particle experiences at any instant in time is:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{9}$$

This is called the *Lorenz Force*. The equation of motion of the charged particle is given by Newton's Second Law:

$$\begin{aligned} m\frac{d\vec{v}}{dt} &= \sum \vec{F} \\ m\frac{d\vec{v}}{dt} &= q(\vec{E} + \vec{v} \times \vec{B}) \end{aligned} \tag{10}$$

This is a tricky looking equation of motion! Let's consider a simpler case in which there is no electric field and the magnetic field is constant in magnitude, pointing in the positive x-direction. We can say that:

$$\vec{B} = B\,\hat{\boldsymbol{x}} \tag{11}$$

Imagine that a charged particle with mass m, positive charge q, and speed v is traveling in the positive y-direction. The velocity of the particle is:

$$\vec{v} = v\,\hat{\boldsymbol{y}} \tag{12}$$

Figure 7 shows a diagram of the problem. The force on the particle is:

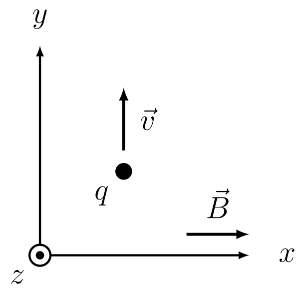

Figure 7: Schematic of a charged particle traveling through a region with constant magnetic field.

$$\begin{aligned} \vec{F} &= q(\vec{v} \times \vec{B}) \\ &= q(v\,\hat{\boldsymbol{y}}) \times (B\,\hat{\boldsymbol{x}}) \\ &= qvB(\hat{\boldsymbol{y}} \times \hat{\boldsymbol{x}}) \\ &= -qvB\hat{\boldsymbol{z}} \end{aligned} \tag{13}$$

Note that $\hat{\boldsymbol{y}} \times \hat{\boldsymbol{x}} = \hat{\boldsymbol{z}}$. Check out this **YouTube Video** for more details on the cross products of unit vectors!

The magnetic force pulls the particle in the negative z direction, which, in this case, is into the page. Remember that the speed of the particle remains constant, only the direction of its velocity changes. As the particle moves into the page, the direction of the force will change since the direction of the velocity changed. Ultimately the particle will orbit around the magnetic field line at a particular radius, as shown in Figure 8.

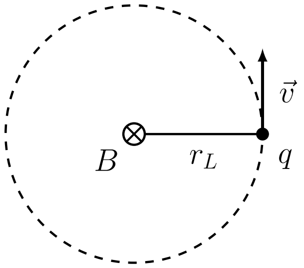

Figure 8: A positively charged particle orbiting counter-clockwise around a magnetic field line.

What is the radius of the particle's orbit? We can use Newton's Second Law in which the magnitude of the Lorenz force is balanced by the centripetal acceleration of the particle:

$$m\frac{v^2}{r} = qvB \tag{14}$$

Now we can solve for the radius, which is called the Larmour radius and is denoted by r_L:

$$r_L = \frac{mv}{qB} \tag{15}$$

Any moving charged particle in a magnetic field will circulate around the field lines. In general, the motion may not be exactly circular and will depend on the distribution of the magnetic field, the presence of an electric field, and the presence of other forces.

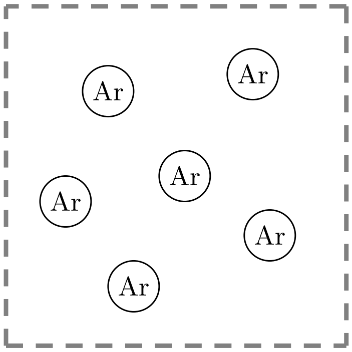

Figure 9: A box of neutral gas composed of Argon atoms.

INTRODUCTION TO PLASMA PHYSICS

What is a Plasma?

For the purposes of this course, a *plasma* is an *ionized gas*.

> DEFINITION 10.1 A gas in which some fraction of the atoms or molecules are ionized is called an **ionized gas**. An ionized atom (or molecule) has one or more of its electrons removed.

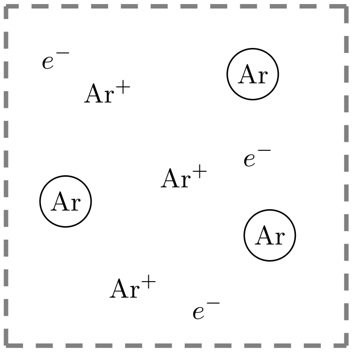

Figure 10: A box of ionized gas composed of neutral Argon atoms, singly charged Argon ions, and free electrons.

A plasma has three distinct particle populations: neutral particles, ionized particles, and free electrons. In contrast, a gas only has one particle population: neutral particles. Figures 9 and 10 show the difference between a gas and an ionized gas.

Plasmas are really interesting because they move collectively. This is because the ions and electrons are charged particles that exert electric forces on each other. The motion of charged particles in a plasma can also generate magnetic fields, which, in turn, affect the motion of the charged particles. In contrast, the atoms or molecules in a gas move independently, in straight lines, because they don't exert forces on each other.

How Do We Make Plasmas?

There are two common methods used to ionize a gas for space propulsion applications:

1. **Electron bombardment** - A cathode is used to generate free electrons, which are accelerated to high speeds using an electric field. The fast electrons collide with neutral atoms or molecules, ionizing them in the process.
2. **Radiofrequency ionization** - Radio waves of the correct frequency can be used to ionize a gas.

Both methods require electrical energy to generate a plasma. The required amount of energy depends on the gas properties.

One of the goals in designing a plasma thruster is to minimize the energy required to ionize the gas. The less energy needed to make the plasma, the less energy required of the power supply, and thus the smaller (meaning less massive) the power supply. Every atom and molecule has its own unique *first ionization potential*. Ultimately we want to choose a gas that has a low first ionization potential, but we need to be careful because we also need to choose a gas that has a high *second ionization potential*.

DEFINITION 10.2 The energy required to remove the outermost electron from a neutral atom or molecule is called the **first ionization potential**.

DEFINITION 10.3 The energy required to remove the outermost electron from a singly ionized atom or molecule is called the **second ionization potential**.

Essentially, we want to create a plasma that has lots of singly charged ions and very few doubly charged ions. This is because the propulsive efficiency of a thruster with only singly charged ions is much higher than that of a thruster with a mix of doubly and singly charged ions. We'll study this in more detail in our lab!

Finally, we want to choose an atomic gas instead of a molecular gas. Atomic gases can only absorb energy through the excitation of their electrons. This means that exciting an atom will efficiently lead to ionization. Molecular gases, on the other hand, can also absorb energy through the excitation of their rotational and vibrational modes.

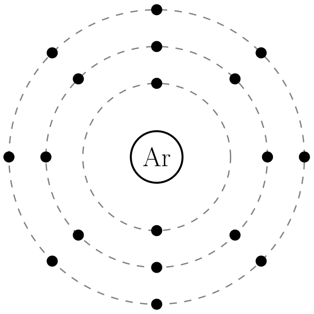

Figure 11: The electron configuration of a neutral Argon atom.

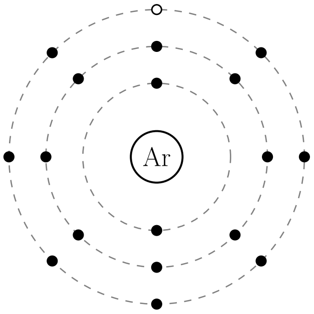

Figure 12: The electron configuration of a singly ionized Argon atom.

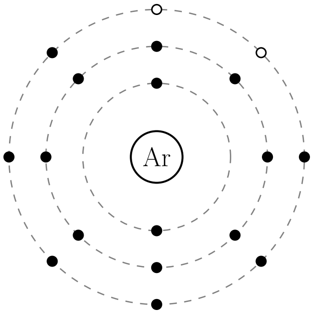

Figure 13: The electron configuration of a doubly ionized Argon atom.

Molecular rotational and vibrational modes are dense and can absorb a lot of energy before an outer electron gains enough energy to leave the molecule. This means that significant energy is wasted on exciting the molecule, instead of the electron, which has zero propulsive benefit. To maximize the efficiency of ionization, atomic gases are used almost exclusively in plasma thrusters.

Noble gases, such as xenon and argon, are most commonly used in plasma thrusters. Their full electron shells prevent them from readily forming molecules, so they remain as single atoms in a gaseous state. They don't have the lowest first ionization potentials, but they do have very high second ionization potentials. Noble gases are inert, meaning that they don't readily react with other chemical substances. This makes them suitable for long term storage and use in an electric thruster. Table 1 shows properties of the propellants commonly used in plasma thrusters.

	Mercury	Argon	Xenon
First Ionization Potential (eV)	10.43	15.8	12.13
Second Ionization Potential (eV)	29.2	27.6	33.3
Atomic Mass (amu)	200.6	39.9	131.3

Table 1: First and second ionization potentials for common propellants used in gridded ion thrusters.

Note that mercury is not a noble gas. Mercury is stored as a liquid and is vaporized before being injected into the ionization chamber.

How Do We Characterize Plasmas?

In this section, we'll investigate some of the unique behaviors of plasmas and learn about some of the parameters used to characterize them. The particles in a plasma are always in motion, just like the particles in a gas. If a pure gas is at a particular temperature, T, the mean speed of the particles is given by the following equation:

$$\bar{c} = \sqrt{\frac{8k_BT}{\pi m}} \tag{16}$$

where $k_B = 1.38 \times 10^{-23}$ $\text{m}^2\text{kg}/\text{s}^2\text{K}$ is Boltzmann's constant and m is the mass of a particle.

A plasma has three constituents: neutral particles, positive ions, and electrons. For now, we will assume that all three constituents are at the same temperature. The particles will have different speeds according to their masses. Since ions and neutral particles have approximately the same mass (the ion is lighter by the mass of one electron, which is a very small amount!), we can say that the mean speed of the ions is equal to the mean speed of the neutral particles. The mean speed of the ions and neutrals is given by:

$$\bar{c}_n = \bar{c}_i = \sqrt{\frac{8k_B T}{\pi m_n}} \qquad (17)$$

where $\bar{c}_n$ is the mean speed of the neutral particles, $\bar{c}_i$ is the mean speed of the ions, and m_n is the mass of a neutral particle. The mean speed of the electrons, $\bar{c}_e$, is given by the following expression:

$$\bar{c}_e = \sqrt{\frac{8k_B T}{\pi m_e}} \qquad (18)$$

where m_e is the mass of an electron.

Electrons are 1000 times lighter than protons. If we consider a plasma made from Xenon gas, the electrons are 100,000 times lighter than the ions! This means that the electrons have a mean speed that is over 100 times faster than the neutral particles and ions. You can think of the ions and neutral particles as large, slow moving spheres and the electrons are a fast-moving swarm of tiny bees!

The difference in mean speeds leads to some very interesting behavior in plasmas. Remember that, on the whole, plasmas have net zero charge. For every positively charged ion, there is a negatively charged electron to balance it. However, imagine zooming in on a small region of the plasma. The electrons move so quickly, that they might clear out of a region, leaving the slow-moving positive ions behind. This creates a region with a net positive charge and thus a net electric field.

The electric field will attract the electrons back into the region. Because the electrons are fast-moving, they will be quick to neutralize the positive charge of the ions. In fact, they might overcompensate and create a net negative charge. The negative charge from the extra electrons in the region will repel electrons, while attracting positive ions. Eventually the region will have a net positive charge after the electrons leave and the ions enter. We've come full circle! The cycle repeats again and again. Small regions of the plasma oscillate between a net positive charge and a net negative charge. The oscillations have a particular frequency, called the *plasma frequency*, which is denoted by ω_p:

$$\omega_p = \sqrt{\frac{e^2 n_e}{m_e \epsilon_0}} \tag{19}$$

where e is the fundamental unit of charge, n_e is the number density of electrons, m_e is the mass of the electrons, and ϵ_0 is the permittivity of free space. Notice that the plasma frequency doesn't depend on the ion mass. This is because the electrons are the fast-moving particles that drive this oscillatory behavior, not the ions.

The plasma frequency is a really interesting property because it is the minimum frequency of an electromagnetic wave that can pass through a plasma. Consider the ionosphere, which is a low-density plasma layer that surrounds the Earth. The plasma frequency of the ionosphere is typically in the megaHertz range, which is a bit higher than the radio frequencies that radio stations use. This means that electromagnetic waves generated by a radio station antenna will bounce off the lower surface of the ionosphere and travel back down to Earth's surface, hundreds of miles away from the radio station. If the radio waves had a higher frequency than the plasma frequency of the ionosphere, they would travel through it and head towards outer space and radio listeners hundreds of miles of away wouldn't be able to hear the broadcast!

We can also estimate the size of the non-neutral regions in a plasma, shown in Figure 14. This is given by the *debye length*:

$$\lambda = \sqrt{\frac{\epsilon_0 k_B T_e}{e^2 n_e}} \tag{20}$$

where T_e is the temperature of the electrons. Just like the plasma frequency, the size of the non-neutral regions is driven by the properties of the electrons, not the ions.

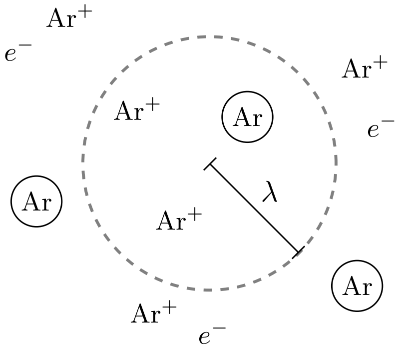

Figure 14: The size of a non-neutral region in a plasma is given by the debye length, which is denoted by λ.

The plasma frequency tells us how quickly plasma respond to charge imbalances. The debye length tells us the largest possible size of a non-neutral region in a plasma. These charge imbalances come from the natural oscillatory motion of the plasma. They can also be driven by external electric and magnetic fields.

Finally, let's consider what happens when a plasma comes into contact with a metallic surface. At the instant the metallic surface is exposed to the plasma, the fast-moving electrons collide with the surface, charging it negatively. The negative charge creates an electric field that attracts ions and repels low energy electrons. Only a small fraction of the electrons have enough energy to reach the metal surface. Eventually, equilibrium is reached and a non-neutral charge layer forms between the plasma and the metallic surface. This non-neutral layer is called a "sheath" and is illustrated in Figure 15.

Within the sheath, the electric field generated by the negatively charged metallic surface can be felt by the ions. Beyond the sheath, the plasma effectively shields the charged surface, and no electric field is felt by the ions. The sheath has a thickness on the order of the debye length and is critical parameter for designing the grids on an ion thruster, which we'll learn about next lecture!

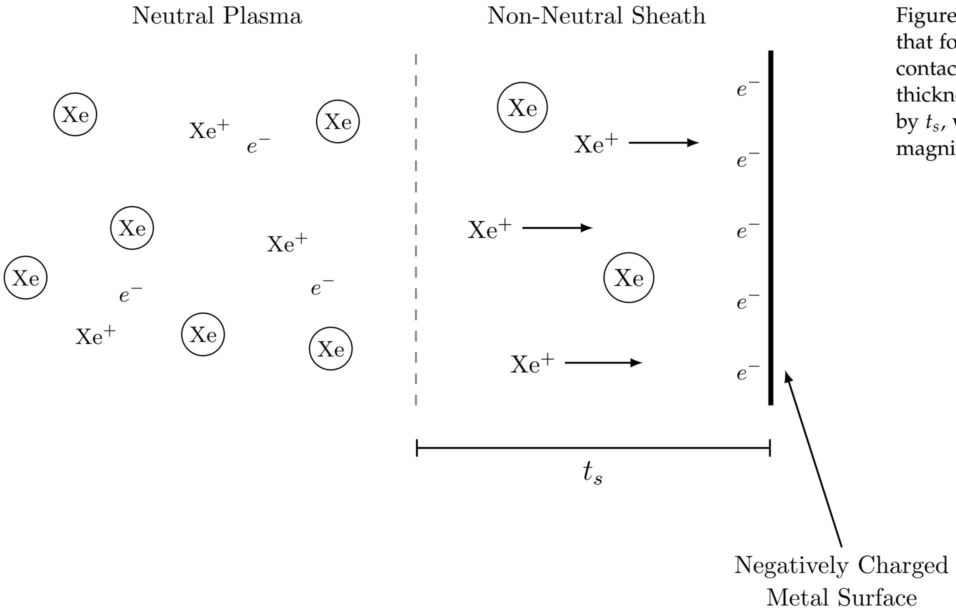

Figure 15: Schematic of a plasma sheath that forms when a plasma comes into contact with a metallic surface. The thickness of the sheath layer is denoted by t_s, which is on the same order of magnitude as the debye length.