INTRODUCTION TO ROCKET PROPULSION

L7: Chemical Rocket Performance

How do we quantify the performance of chemical rockets?
In this lecture, we will derive the thrust equation of a chemical rocket. We'll investigate how pressure forces arise in chemical rocket engines and quantify how they contribute to the thrust force. Using the thrust equation, we'll calculate the specific impulse of a chemical rocket. We'll also learn how the thrust and specific impulse change with altitude. Finally, we will draw the shape of the exhaust plume for different pressure conditions.

Figure 1: The exhaust plume of the Saturn V launch vehicle. The exhaust is *under-expanded*, which is why the plume extends far beyond the exit area of the rocket nozzle. We'll learn about why this happens later in the lecture!

LEARNING GOALS:

1. Derive the equation for thrust of a chemical rocket, which includes the pressure forces acting on the engine.
2. Calculate the specific impulse of a chemical rocket.
3. Describe how thrust and specific impulse change with altitude.
4. Draw the streamlines of the exhaust gas leaving the rocket nozzle for various pressure conditions.

CHEMICAL ROCKET DEFINITIONS

In order to analyze chemical rocket engines, we will need to define several quantities: *chamber pressure*, *chamber temperature*, *exhaust pressure*, and *ambient pressure*. Figure 3 shows these quantities labeled at their corresponding locations in the rocket engine. Temperature and pressure are *state variables*, which are used to define the state of a system. In the case of a chemical rocket engine, the system is the combustion gases flowing through the engine. We can completely describe the state of the gaseous products at any point within the rocket engine if we know the values of two state variables. Density is also a state variable, but we typically use temperature and pressure to analyze chemical rockets.

DEFINITION 7.1 A **state variable** is used to mathematically describe the state of a system. Some common state variables include pressure, temperature, density, and mass. State variables can only be measured when the system is at equilibrium.

DEFINITION 7.2 The **chamber pressure** is the pressure of the gaseous combustion products inside the combustion chamber. It is denoted by p_c and is approximately equal to the pressure of the liquid propellants before they are injected into the combustion chamber. Chamber pressure is a design variable that can be chosen somewhat flexibly.

DEFINITION 7.3 The **chamber temperature** is the temperature of the gaseous combustion products immediately after the combustion reaction. It is also called the *combustion temperature* and is denoted by T_c. The chamber temperature is a function of the oxidizer-to-fuel ratio and the chamber pressure. The chamber temperature is highest for stoichiometric reactions.

DEFINITION 7.4 The **exhaust pressure** is the pressure of the gaseous combustion products at the nozzle exit. At this point the gaseous combustion products become "exhaust". The exhaust pressure is denoted by p_e and is a function of the chamber pressure and the shape of the nozzle, namely that ratio of the exit area to the throat area.

DEFINITION 7.5 The **ambient pressure** is the pressure of the air surrounding the rocket engine. It is also called *atmospheric pressure* and is denoted by p_a. The atmospheric pressure is a function of the altitude above Earth's surface.

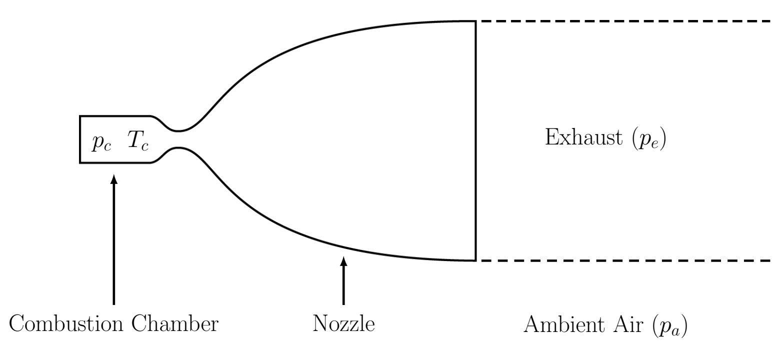

Figure 2: Diagram of a bi-propellant chemical rocket with state variables labeled.

CHEMICAL ROCKET THRUST

In Lecture 2, we derived the following equation for rocket thrust:

$$F = \dot{m}c \tag{1}$$

where $\dot{m}$ is the mass flow rate and c is the exhaust velocity. In deriving this equation, we assumed that there are no external forces acting on the rocket. However, this isn't true for chemical rockets! In chemical rockets, we need to account for pressure forces.

Pressure Force

There are two external forces acting on a chemical rocket engine due to the pressure of the surrounding gas. The first force arises from the pressure exerted by the exhaust gases that leave the rocket. The second force comes from the pressure exerted by the atmosphere surrounding the rocket engine.

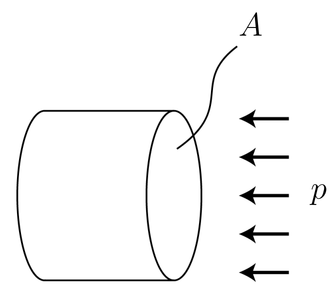

Figure 3: Atmospheric pressure forces acting on the rightmost surface of the cylinder.

What is *pressure force*? It's the force exerted by a gas on a surface due to the pressure of the gas. Imagine an object with a flat surface, like the cylinder shown in Figure 3. Assume that the object is surrounded by a gas with a pressure, p, which has units of Newtons per meter squared. To obtain the force exerted by the air on the rightmost surface of the cylinder, we need to multiply the pressure by the area of the surface. We can express the magnitude of the pressure force, F_p, using the following equation:

$$F_p = pA \tag{2}$$

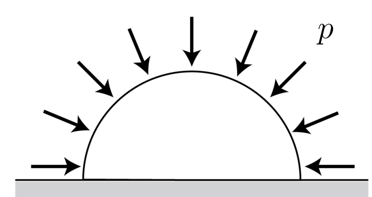

Figure 4: Atmospheric pressure forces acting on a water droplet placed on a flat surface. The pressure force always acts normal, meaning perpendicular, to the surface of the droplet.

where p is the pressure and A is the area of the flat surface. The force acts perpendicular to the flat surface, pushing into it. If the surface were curved, the pressure force at a specific point on the surface would also act normal to the surface. The perpendicular direction changes along the curved surface, as shown in Figure 4. To obtain the net force on the entire surface, we would need to integrate the local pressure force over the entire surface area while taking into account the direction of the forces.

Pressure Forces on a Chemical Rocket

Let's take a closer look at the pressure forces acting on a chemical rocket. Remember that there are two types of pressure forces: exhaust pressure and atmospheric pressure. We'll start by considering the force from the exhaust gases.

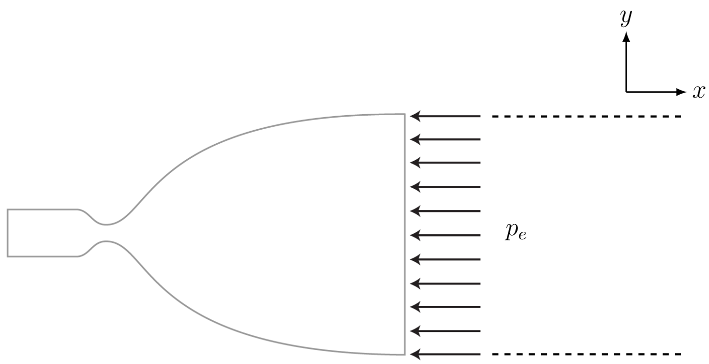

Figure 5: Exhaust pressure forces acting on a chemical rocket engine.

Figure 5 shows a diagram of a chemical rocket engine, viewed from the side. The exhaust gases exit the rocket through the nozzle in the positive x-direction. The dashed lines show the boundary between the exhaust gases and the ambient air around the rocket.

Let's consider the exhaust gases at the exit plane of the nozzle. We'll assume that the pressure of the exhaust is uniformly distributed across the surface of the exit plane. The pressure force exerted by the exhaust on the exit plane of the nozzle is F_e:

$$F_e = -p_e A_e \tag{3}$$

where p_e is the pressure of the exhaust, and A_e is the area of the nozzle exit plane. The negative sign indicates that the pressure force acts in the negative x-direction. Since the thrust force on the vehicle also points in the negative x-direction, the exhaust pressure force increases the thrust.

Now let's consider the pressure of the ambient air. The atmospheric pressure acts over all of the surfaces of the rocket engine, except for the exit plane of the nozzle. Figure 6 shows the atmospheric pressure forces acting on the rocket engine. Notice that many of the arrows will cancel out. There is no net force in the y-direction because every arrow that points up is canceled out by an arrow that points down.

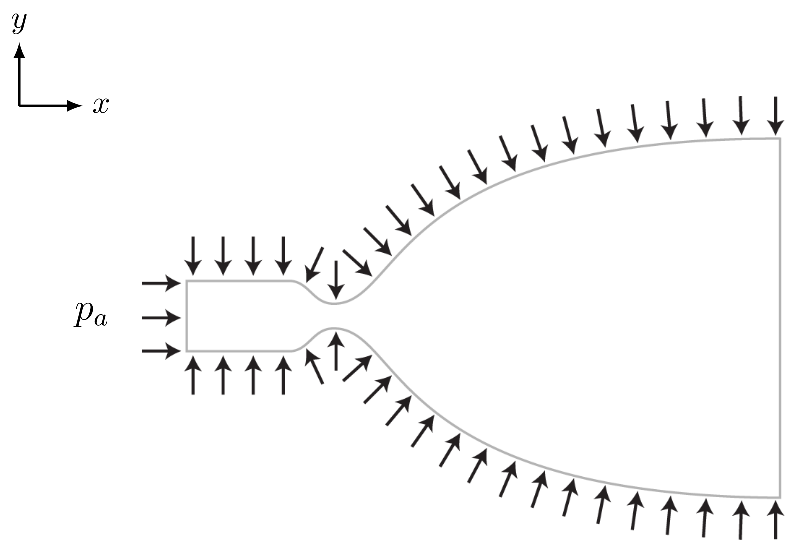

Figure 6: Ambient air pressure forces acting on an operating chemical rocket. The pressure forces of the exhaust gases and thrust force are not drawn.

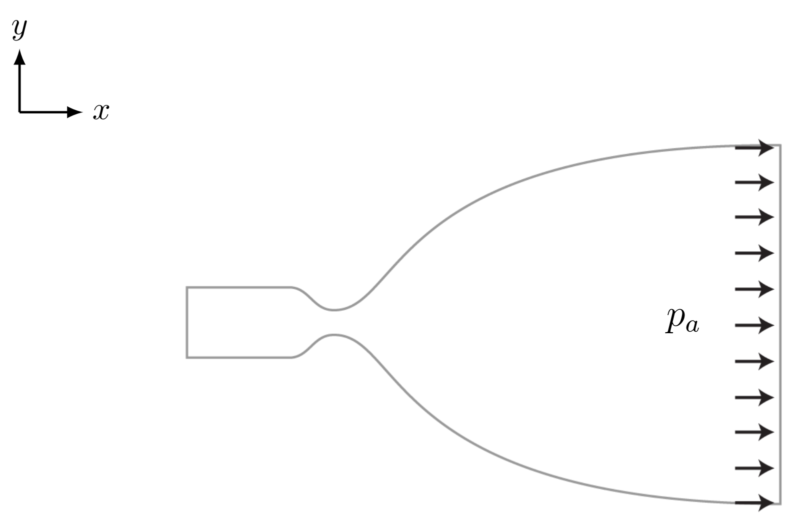

Figure 7: Net ambient air pressure forces acting on an operating chemical rocket. The pressure forces of the exhaust gases and thrust force are not drawn.

There is a net force in the x-direction, shown in Figure 7, because the arrows that point to the right aren't canceled by arrows pointing to the left. Notice that if the atmospheric pressure also acted along the nozzle exit plane, there would be arrows pointing to the left and the net force in the x-direction would be zero. This situation happens when the rocket is not operating, which is shown in Figure 8.

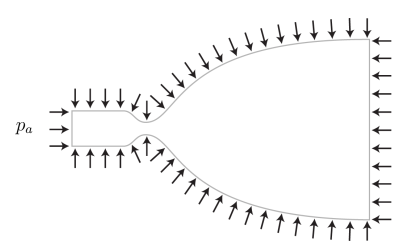

Figure 8: Ambient air pressure forces acting on a chemical rocket when it is not operating. In this case, the net forces in the x and y directions are zero.

Once the rocket turns on, the exhaust pressure acts on the exit plane, which creates an imbalance of the atmospheric pressure forces. The net force in the x-direction due to atmospheric pressure forces on the rocket is F_a:

$$F_a = p_a A_e \tag{4}$$

where p_a is the pressure of the surrounding atmosphere. This force acts in the positive x-direction, which means that it decreases the total thrust.

The net pressure force acting on a chemical rocket, F_p, is the sum of the exhaust pressure force and the atmospheric pressure force:

$$\begin{aligned} F_p &= F_a + F_e \\ &= p_a A_e - p_e A_e \\ &= -(p_e - p_a) A_e \end{aligned} \tag{4}$$

The pressure force acting on the rocket in the direction of the thrust, in this case the negative x-direction, is $(p_e - p_a)A_e$. Therefore, if the exhaust pressure is greater than the ambient pressure, the net pressure force on the rocket will increase the thrust.

Net Force Acting on a Chemical Rocket

Now that we have found the pressure force acting on a chemical rocket, we can determine the thrust. In Lecture 2, we found that the sum of the forces on the rocket in the x-direction is equal to the change in momentum of the exhaust:

$$\sum F_x = \dot{m} u_e \tag{5}$$

where u_e is the velocity of the exhaust at the exit plane of the nozzle. Figure 9 shows the net forces acting on the rocket. The sum of the forces in the x-direction is:

$$\sum F_x = F + (p_a - p_e) A_e \tag{6}$$

where F is the thrust. Now we can use equations 5 and 6 to solve for the thrust:

$$F = \dot{m} u_e + (p_e - p_a) A_e \tag{7}$$

The first term in the thrust equation is the momentum flux, which is the rate at which the exhaust carries momentum away from the vehicle. The second term is the net pressure force on the rocket. Notice that if the atmospheric pressure is greater than the exhaust pressure, the net pressure force will be negative.

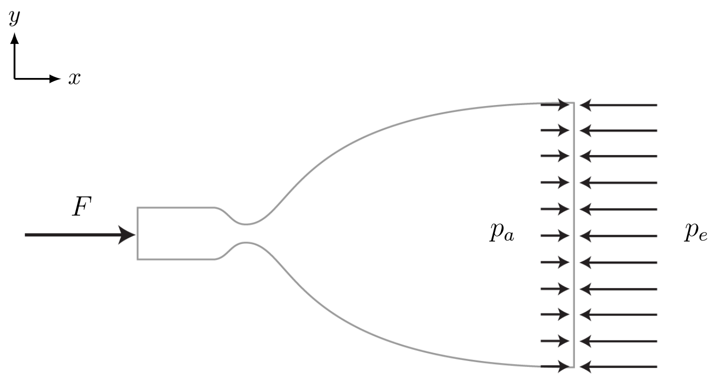

Figure 9: Net forces acting on an operational chemical rocket. F is the equal and opposite reaction force that pushes the vehicle in the negative x-direction.

We can define the *effective exhaust velocity*, c, of a chemical rocket by setting the thrust equal to $\dot{m}c$:

$$F = \dot{m}c = \dot{m}u_e + (p_e - p_a)A_e \tag{8}$$

Solving for c, we obtain the following expression:

$$c = u_e + \frac{(p_e - p_a)A_e}{\dot{m}} \tag{9}$$

The effective exhaust velocity is the true exhaust velocity plus the pressure thrust divided by the mass flow rate.

CHEMICAL ROCKET SPECIFIC IMPULSE

Now that we have an expression for the thrust, we can estimate the specific impulse. Let's assume the thrust and specific impulse, which is given by the following equation:

$$I_{sp} = \frac{F}{\dot{m}g} \tag{10}$$

We can substitute equation 7 for the thrust force in the numerator:

$$\begin{aligned} I_{sp} &= \frac{\dot{m}u_e + (p_e - p_a)A_e}{\dot{m}g} \\ &= \frac{u_e}{g} + \frac{(p_e - p_a)A_e}{\dot{m}g} \\ &= \frac{c}{g} \end{aligned} \tag{11}$$

We can write the specific impulse in a compact form using the effective exhaust velocity.

CHEMICAL ROCKET PERFORMANCE VS. ALTITUDE

The thrust and specific impulse of a chemical rocket depend on the atmospheric pressure, which changes as a function of the altitude above Earth's surface. For example, the pressure at sea level is 1 atm, whereas at 10 km the pressure is 0.26 atm. Closer to 100 km the atmospheric pressure drops to essentially zero. How does this affect the performance of a chemical rocket as it lifts off the launch pad and ascends to space?

Generally speaking, the pressure of the exhaust is constant and is set by the design of the rocket. Therefore, the only quantity that changes with altitude is the atmospheric pressure. Looking at equation 7, we can notice that the pressure thrust will increase as atmospheric pressure decreases. This means that the total thrust will increase as well. Therefore, we should expect the thrust of the rocket to increase as the rocket ascends to space.

What about the specific impulse? The specific impulse depends on the atmospheric pressure in the same way that the thrust does. If we assume that the mass flow rate and exhaust pressure remain constant, then we can say that the effective exhaust velocity will increase as the atmospheric pressure decreases. Thus, we should expect the specific impulse of a chemical rocket to increase as the altitude of the rocket increases.

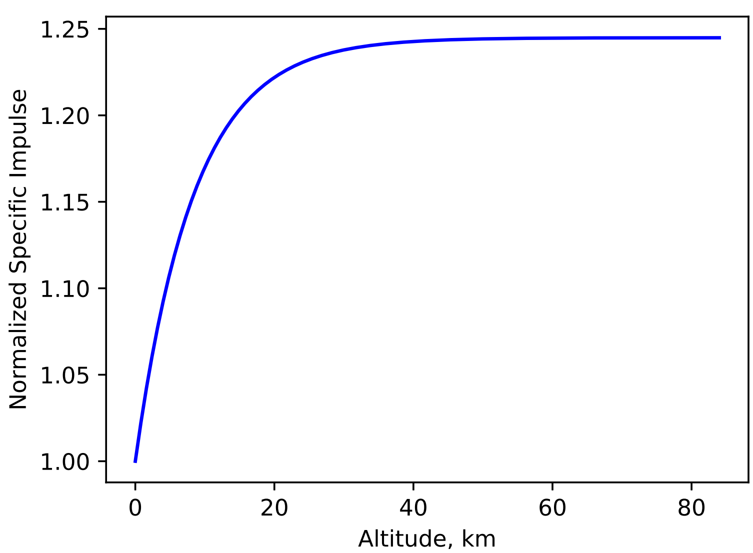

Figure 10: The specific impulse of a rocket engine, normalized by the specific impulse at sea-level, as a function of altitude above the Earth. The specific impulse increases with altitude because the pressure thrust increases as the atmospheric pressure decreases.

Exhaust Streamlines Under Various Pressure Conditions

The shape of the exhaust leaving the nozzle depends on the pressure conditions. In Figure 3, we drew the exhaust coming straight out of the nozzle. This happens when the ambient pressure is equal to the exhaust pressure. In this case, the exhaust is *perfectly expanded*. The streamlines for this condition are shown in Figure 11. A streamline is the path that a gas molecule follows as it leaves the nozzle.

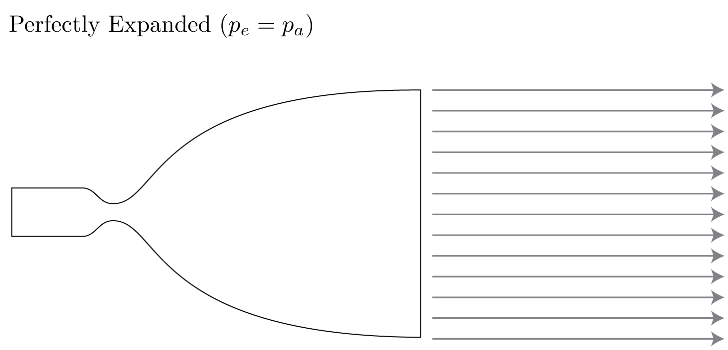

Figure 11: Perfectly expanded exhaust gas streamlines.

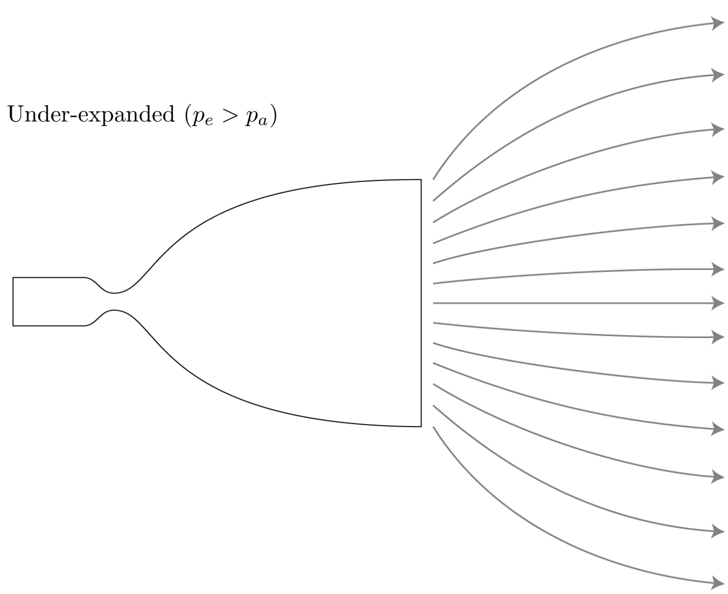

Figure 12: Under-expanded exhaust gas streamlines.

What happens when the exhaust pressure is greater than the ambient pressure? High pressure gas always wants to expand into lower pressure gas. Therefore, the high pressure exhaust will expand outwards into the low pressure ambient air. The exhaust will expand until its pressure decreases enough to equal the ambient pressure. We call this type of exhaust *under-expanded* because the nozzle does not fully expand the gas to a pressure equal to the atmospheric pressure. This condition is shown in Figure 12.

What happens when the exhaust pressure is lower than the ambient pressure? In this case, the high pressure ambient air will expand into the low pressure exhaust. The exhaust gases are forced into a region smaller than the exit area of the nozzle. This type of exhaust is *over-expanded* because the nozzle expands the gas too much. This condition is shown in Figure 13.

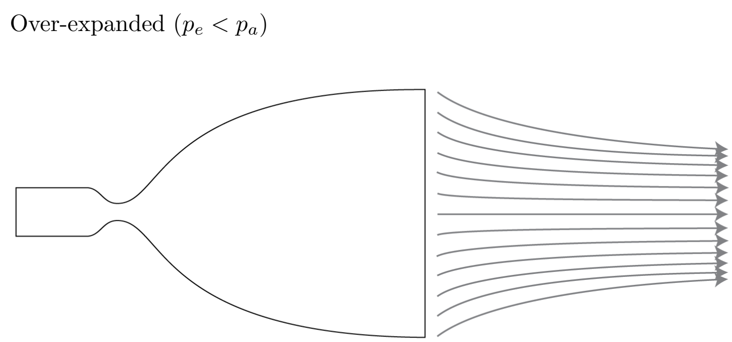

Figure 13: Over-expanded exhaust gas streamlines.

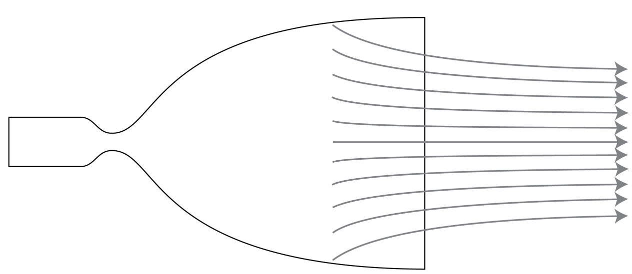

Figure 14: Over-expanded exhaust gas streamlines with flow separation.

We need to be careful about overexpanding the exhaust because it can lead to *flow separation.* Under ideal conditions, the exhaust gases stay in contact with the internal walls of the nozzle. This ensures that the flow area of the exhaust as it leaves the engine is equal to the exit area of the nozzle. However, if the exhaust pressure is too low, the exhaust gases can separate from the internal walls of the nozzle. This leads to reduced performance because the exit area of the exhaust is less than the nozzle exit area, which is shown in Figure 14. Flow separation typically occurs when:

$$p_e \leq 0.4 p_a \tag{12}$$

As long as we design the rocket nozzle to keep the exhaust pressure above $0.4 p_a$, then flow separation should not occur. We will learn about nozzle design in the next lecture!